2023

Time - 3 hours

Full Marks - 80

*Answer **all parts** as per instructions.*
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

PART – I

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) If the repeated limits of f(x, y) exist and are equal, then the simultaneous limit of f(x, y) exists. Write True or False.

(b) Let $z = f(x, y)$, then $dz =$ _____

(c) Grad ϕ defines a _____ field.

(d) The points at which $df =$ _____ are called stationary points.

(e) Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$. Then find curl $\vec{F}$.

(f) A vector $\vec{F}$ is solenoidal if _____.

(g) If $x = r\cos\theta$, $y = r\sin\theta$, then $dx\,dy =$ _____.

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(h) The integral $\int_1^5 \int_2^4 f(x, y)dx\,dy$ integrates f over the rectangle defined by _____ ≤ x ≤ _____ and _____ ≤ y ≤ _____.

(i) Write the cylinderical polar coordinates of (x, y, z).

(j) What is the work done by a force $\vec{F}$ in moving a particle along a curve r(t) ?

(k) A field $\vec{F}$ is conservative if _____.

(l) Write the vector form of Green's theorem.

PART – II

2. Answer any eight of the following questions. [2 × 8

(a) Find the repeated limits $(x, y) \to (0, 0)$ of $f(x,y) = \frac{2xy^2}{x^2 + y^4}$.

(b) If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$, then find dz/dt.

(c) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$.

(d) Find the stationary points of $f(x, y) = y^2 + x^2y + x^2$.

(e) If $\vec{a}$ is a constant vector, then find $\text{div}(\vec{r} \times \vec{a})$.

(f) Evaluate $\iint_R (x + 2y)dx\,dy$ where R = [1, 2 : 3, 5]

(g) Evaluate $\iint_C (x^2 + y^2)dx\,dy$ where C is $x^2 + y^2 = a^2$.

(h) Evaluate the integral by passing over to spherical polar coordinate $\int_0^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} dy \int_0^9 dz$.

(i) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = xy^2\hat{i} + y\hat{j}$ and the curve C is $y^2 = 4x$ in the xy-plane from (0, 0) to (4, 4).

(j) Evaluate by Green's theorem $\int_C (e^x \sin y\,dx + e^{-x}\cos y\,dy)$ where C is the rectangle with vertices (0, 0), (π, 0), $\left(\pi, \frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{2}\right)$.

PART – III

3. Answer any eight of the following questions. [3 × 8

(a) Test the continuity of the function f(x, y) at (0, 0) where

$$f(x,y) = \begin{cases} \frac{x^4 - y^4}{x^4 + y^2} & , \quad (x,y) \neq (0,0) \\ \end{cases}$$

and f(0, 0) = 0

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(b) Find the equation of tangent plane of the surface $xyz = 4$ at the point (1, 2, 2).

(c) Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & , \ x^2+y^2 \neq 0 \\ 0 & , \ x=y=0 \end{cases}$ then find $f_x(0, 0)$.

(d) Evaluate $\iint (x^2 + y^2)dx\,dy$ over the domain bounded by $y = x^2$ and $y^2 = x$.

(e) Verify that the vector $\vec{F} = (6xy + z^3)\hat{i} + 3x^2z\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.

(f) Prove that $\nabla \times (\nabla \times \vec{u}) = \nabla.(\nabla.\vec{u}) - \nabla^2\vec{u}$.

(g) Compute $\iiint xyz\,dx\,dy\,dz$ over a domain bounded by $x = 0, y = 0, z = 0, x + y + z = 1$.

(h) Show that the field $\vec{F} = \cos y\,\hat{i} - x \sin y\,\hat{j} - \cos z\,\hat{k}$ is conservative.

(i) Find the work down when a force $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ moves a particle in the xy-plane from (0, 0) to (1, 1).

(j) Evaluate $\int_S (yz\,\hat{i} + zx\hat{j} + xy\hat{k})ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the 1st octant.

PART – IV

Answer all questions.

4. Show that the function f where

$$f(x,y) = \begin{cases} xy\,\frac{x^2-y^2}{x^2+y^2} & , \text{ if } x^2+y^2 \neq 0 \\ 0 & , \text{ if } x=y=0 \end{cases}$$

is differentiable at origin. [7

OR

Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, 2, –1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

5. Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$. [7

OR

Prove that $\nabla.(\vec{u}.\vec{v}) = \vec{u} \times (\nabla \times \vec{v}) + \vec{v} \times (\nabla \times \vec{u}) + (\vec{u}.\nabla)\vec{v} + (\vec{v}.\nabla)\vec{u}$.

6. Evaluate $\iiint z^2\,dx\,dy\,dz$ taken over the region common to the surface $x^2 + y^2 + z^2 = a^2, x^2 + y^2 = ax$. [7

OR

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Compute $\iiint_C xy\,dx\,dy\,dz$ where C is the domain bounded by $z = xy$, $x + y = 1$, $z = 0$ $(z \geq 0)$.

7. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the 1st octant. [7

OR

Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by $x = \pm a$, $y = 0$, $y = b$.

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(f) For any primal and its dual optimum value of the objective function is same. Write true or false.

(g) The solution of a transportation problem with m sources and n destinations is feasible if the number of allocations are _____.

(h) The initial solution of a transportation problem is obtained by _____ would invariably be very near to the optimal solution.

(i) In an assignment problem involving 4 workers and 3 jobs, total number of assignments possible are _____.

(j) For a 2 person zero sum game, the value of game can be _____, _____, _____.

(k) A game is said to be fair if upper value is more than lower value of the game, is true or false.

(l) The graphic approach to the solution of games can be applied when one of the players has two strategies available, is true or false.

PART – II

2. Answer any eight of the following questions. [2 × 8

(a) Define Basic Variable and Basic Solution.

(b) Differentiate between feasible solution and optimum basic feasible solution.

(c) Write the role of Pivot element in simplex table.

(d) State fundamental theorem of duality.

(e) Define Primal problem.

(f) What is shadow price ? How does it relate to Primal ?

(g) What is transportation problem ?

(h) What is an assignment problem ?

(i) Describe a two person zero-sum game.

(j) Explain mixed strategy in game theory.

PART – III

3. Answer any eight of the following questions. [3 × 8

(a) Prove that the set of all feasible solutions of an LPP forms a convex set.

(b) Explain the use of artificial variables in LPP.

(c) What do you mean by two phase method for solving a given LPP ? Why is it used ?

(d) Prove that dual of the dual of a given primal is again a primal.

(e) What is the significance of duality theory in LPP ? Describe the general rules for writing the dual of an LPP.

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(f) Obtain the dual of the LPP

$$\text{Maximize } Z = 2x_1 + 3x_2 + x_3$$

$$\text{subject to constraints : } 4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

(g) Explain North-West corner method to solve transportation problem for an initial solution.

(h) Give the mathematical formulation of an assignment problem.

(i) Define saddle point. Is it necessary that a game should always possess a saddle point ?

(j) Explain the method of finding optimal solutions of two person zero-sum games.

PART – IV

Answer all questions

4. Use simplex method to solve the following LPP [7

$$\text{Maximize } Z = 4x_1 + 10x_2$$

$$\text{subject to constraints : } 2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1 \geq 0, x_2 \geq 0$$

OR

Use Big-M method to

$$\text{Maximize } Z = 6x_1 + 4x_2$$

$$\text{subject to constraints : } 2x_1 + 3x_2 \leq 30$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1 + x_2 \geq 3$$

$$x_1 \geq 0, x_2 \geq 0$$

5. Use duality in solving the LPP : [7

$$\text{Maximize } Z = x_1 - x_2 + 3x_3 + 2x_4$$

$$\text{subject to constraints : } x_1 + x_2 \geq -1$$

$$x_1 - 3x_2 - x_3 \leq 7$$

$$x_1 - x_3 - 3x_4 = -2$$

$$x_j \geq 0 \ (j = 1, 2, 3, 4).$$

OR

One unit product A contributes Rs. 7 and requires 3 units of raw material and 2 hours of labour. One unit of product B contributes Rs. 5 and requires 2 units of raw material and 3 hours of labour. Availability of raw material at present is of 240 units and there are 200 hours of labour.

(a) Formulate it as an LPP.

(b) Write its dual and also its economic interpretation.

(c) Find the shadow prices of the raw material and labour.

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6. Use Vogel's Approximation method to obtain an initial basic feasible solution of the transportation problem [7

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

OR

A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below

	Men			
Tasks	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated one to a man so as to minimize the total man hours ?

7. Solve the game whose pay off matrix is given by [7

Player B

Player A	B_1	B_2	B_3
A_1	1	3	1
A_2	0	−4	−3
A_3	1	5	−1

OR

Solve the following 2 × 2 game graphically

Player B

Player A	B_1	B_2	B_3	B_4
A_1	2	1	0	−2
A_2	1	0	3	2

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PART – I

1. Answer all questions. [1 × 12

(a) Does every vector space contain a zero vector ?

(b) A set consists of a single non zero vector is L.I. or L.D. ?

(c) Every vector space has a finite basis. Write True or False.

(d) Let V and W are finite dimensional vector spaces over F. T is a function from V to W. If $T(x + y) = T(x) + T(y)$ for $x, y \in V$, then T is linear. True or False.

(e) Is zero matrix a diagonal matrix ?

(f) The cancellation property for multiplication of matrices is valid or not ?

(g) Suppose V is a vector space over F. When V is isomorphic to F^n ?

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(h) Is every vector space isomorphic to its dual space ?

(i) Eigen values must be non zero scalars. True or False.

(j) An inner product is a scalar valued function on the set of ordered pairs of vectors.

(k) The adjoint of a linear operator is unique. True or False.

(l) Every normal operator is diagonalizable. Show it.

PART – II

2. Answer any eight of the following questions. [2 × 8

(a) Let W_1 and W_2 are subspaces of a vector space V. Prove that $W_1 \cup W_2$ is a subspace of V if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

(b) Show that the vectors (1, 1, 0), (1, 0, 1) and (0, 1, 1) generate F^3

(c) Let V be a vector space and let $S_1 \subseteq S_2 \subseteq V$. If S_1 is L.D., then prove that S_2 is L.D.

(d) Suppose $T : R^3 \to R$ be the linear transformation defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Find N(T).

(e) Let $T : R^2 \to R^3$ be the linear transformation defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ and β and γ be the standard ordered bases for R^2 and R^3 respectively. Find $[T]_\beta^\gamma$.

(f) Find all the eigen vales of $\begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix}$.

(g) Let V be an inner product space. Prove that for $x, y, z \in V$, $\langle x, y+z \rangle = \langle x, y \rangle + \langle x, z \rangle$.

(h) Let V be an inner product space and S and S_0 be subsets of V. Prove that $S_0 \subseteq S \Rightarrow S^\perp \subseteq S_0^\perp$.

(i) Let T be a linear operator on a finite dimensional inner product vector space V. If T has an eigen value, then T^* has also an eigen value.

(j) Suppose V be a complex inner product space and T be a linear operator on V. Define $T_1 = \frac{1}{2}(T + T^*)$ and $T_2 = \frac{1}{2i}(T - T^*)$. Prove that T_1 and T_2 are self adjoint.

PART – III

3. Answer any eight of the following questions. [3 × 8

(a) Show that (–2, 0, 3) is a linear combination of (1, 3, 0) and (2, 4, –1).

(b) Show that the set $\{1, x, x^2, \ldots\ldots x^n\}$ is L.I. in $P_n(F)$.

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(c) Let V be a vector space and $\beta = (u_1, u_2, \ldots\ldots u_m)$ be a subset of V. Prove that β is a basis for V if each $u \in V$ can be uniquely expressed as a linear comination of vectors of β.

(d) Let V and W be vector spaces and $T : V \to W$ be linear. Prove that N(T) is a subspace of V.

(e) Let V and W be vector spaces and let $T : V \to W$ be linear and invertible. Prove that T^{-1} is linear.

(f) Suppose T be a linear operator on R^2 defined by $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3a-b \\ a+3b \end{pmatrix}$ and let $\beta = \{(1, 1), (1, -1)\}$ and $\beta' = \{(2, 4), (3, 1)\}$ be ordered bases of R^2. Find the change of coordinate matrix that changes β' coordinates into β coordinate.

(g) Let V be an inner product space over F. Prove that for all $x, y \in V$, $\|x + y\| \leq \|x\| + \|y\|$.

(h) Let V be an inner product space and let T and U be linear operators on V. Prove that $(T + U)^* = T^* + U^*$.

(i) Let T and U be self adjoint operators on an inner product space V. Prove that UT is self adjoint if TU = UT.

(j) Let V be an inner product space and T be a normal operator on V. prove that $\|T(x)\| = \|T^*(x)\| \; \forall x \in V$.

PART – IV

Answer all questions.

4. Suppose W_1 and W_2 be subspaces of a vector space V. Prove that V is a direct sum of W_1 and W_2 if and only if each vector in V can be uniquely written as $x_1 + x_2$ where $x_1 \in W_1$, $x_2 \in W_2$. [7

OR

Let V and W be vector spaces and $T : V \to W$ be linear. If V is finite dimensional then prove that nullity(T) + rank(T) = dim(V).

5. Let V and W be finite dimensional vector spaces with ordered bases β and ν respectively. Let $T : V \to W$ be linear. Prove that T is invertible if and only if $[T]_\beta^\nu$ is invertible. Further more $[T^{-1}]_\nu^\beta = ([T]_\beta^\nu)^{-1}$. [7

OR

Let $T : R^2 \to R^3$ be defined by $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Suppose β be the standard basis for R^2 and $\nu = \{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$. Compute $[T]_\beta^\nu$. If $\alpha = \{(1, 2), (2, 3)\}$, compute $[T]_\alpha^\nu$.

6. Let T be a linear operator on a finite dimensional vector space V and let f(t) be the characteristic polynomial of T. Prove that $f(T) = T_0$, the zero transformation ? [7

OR

P.T.O.

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Apply Gram-Schmidt process to the given subset S of the inner product space V to obtain an orthogonal basis for span (S). Then normalize the vectors in this basis to obtain orthonormal basis β for span(S), where $V = R^3$, $S = \{(1, 0, 1), (0, 1, 1), (1, 3, 3)\}$ and $x = \{1, 1, 2\}$.

7. Let V be a finite dimensional vector space and let T be a linear operator on V. Prove that there exists a unique function $T^* : V \to V$ such that $\langle T(x), y\rangle = \langle x, T^*(y)\rangle$ for $x, y \in V$ and T^* is linear. [7

OR

Find the minimal solution of the system

$$x + 2y + z = 4$$

$$x - y + 2z = -11$$

$$x + 5y = 19$$

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